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Finite volume modeling of the non-isothermal flow of a non-Newtonian fluid in a rubber's extrusion die

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ABSTRACT

Non-isothermal flow of a non-Newtonian fluid is the most complex and important problem in the rubber's extrusion process. In this way, the aim of this work is to describe the computer modeling of the laminar flow through a nozzle by the finite volume method (FVM). The basis of the general mathematical treatment of flow processes are the balance equations for mass, momentum and energy. The flow can be fully described only when the velocity vector and the thermodynamic data as pressure, density and temperature are known at any time and at any point of the flow. To determine these quantities the conservation equations are combined with the constitutive equations which describe the correlations between parameters relating to motion and kinetics on the one hand and between the individual thermodynamic parameters on the other hand. Extrusion heads for the fabrication of rubber profiles are up to now designed on the basis of empirical knowledge of the non-linear inelastic flow behavior involving the heat transfer. The liquid rubber exhibits a shear rate and temperature-dependent viscosity, with 'shear thinning', that is, decreasing viscosity with increasing shear rate and temperature. We have taken the power-law model in order to simulate this rubber's extrusion process. The mathematical model has the form $\mu(t) = K(T)I_2^{\{n(T)-1\}/2}$ where T, μ, I_2, n and K are termed the temperature, dynamic viscosity, the second invariant of the rate of deformation tensor, the power-law index and the consistency, respectively. These last two parameters were obtained at different temperatures from experimental tests and used in the computational simulation. Finally we have modeled the extrusion process for a type of nozzle, H810, in order to calculate the outlet velocity and temperature distribution of the rubber and conclusions are exposed.

1. Introduction

When extruding elastomeric materials such as rubbers, the dimensions of the product are essentially determined by the geometry of the extrusion die. It must be also taken into consideration that the rheological and thermodynamics processes in the die as well as any stretching processes have a decisive effect on the quality of the extruded semi-finished products. In order to design the extrusion die from a process engineering point of view, it is necessary to take into account the flow, deformation, and temperature relationships of the production line. The memory effects of rubbers are not important, so that they can be considered 'inelastic fluids' [1].These materials are generalizations of the Newtonian fluid.

Factors determining the rubber flow besides shear rate γ and shear stress τ are: the temperature T, the hydrostatic pressure p, the molecular weight as well as additives. On the one hand, the effect of temperature on the viscosity is considerably more pronounced at low shear rates, particularly in the range of the zero shear viscosity. On the other hand, the viscosity curves in the diagram are shifted with the temperature, but their shape remains the same.

2. Constitutive equations of the like-rubber non-Newtonian fluid

In this work, we have modeled the behaviour of the rubber in liquid state according to the power-law model, so that the viscosity μ has the form [2,3]:

$$\mu(t) = K(T) \cdot I_2^{(n(T)-1)/2}, \tag{1}$$

where n and K are the temperature-dependent power-law index and consistency, respectively, and I_2 is the second invariant of the

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Table 1
Power-law parameters

Parameter	Lower	Upper
Viscosity μ (Pa s)	26654	180546
Temperature (K)	298	325

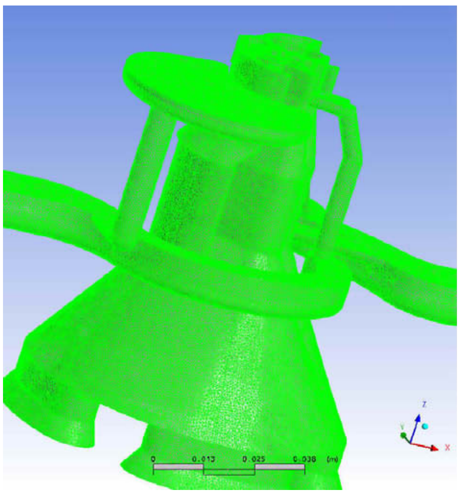

Fig. 1. Detail of the volume finite mesh for the extrusion die H810.

rate of deformation tensor [4]. The liquid rubber inside the die has an index $n < 1$ (pseudoplastic) and its admissible interval of shear rate ranges from 10 to 100 s^{-1}. In order to tackle this complex non-linear problem, we have carried out a linearization of Eq. (1):

$$\mu(T) = \mu_{\text{lower}} + \frac{(\mu_{\text{upper}} - \mu_{\text{lower}})}{T_{\text{upper}} - T_{\text{lower}}} \cdot (T - T_{\text{lower}}), \tag{2}$$

where μ_{lower} and μ_{upper} are the minimum and maximum values of the viscosity corresponding to the minimum and maximum temperatures T_{lower} and T_{upper}, respectively, in the extrusion process.

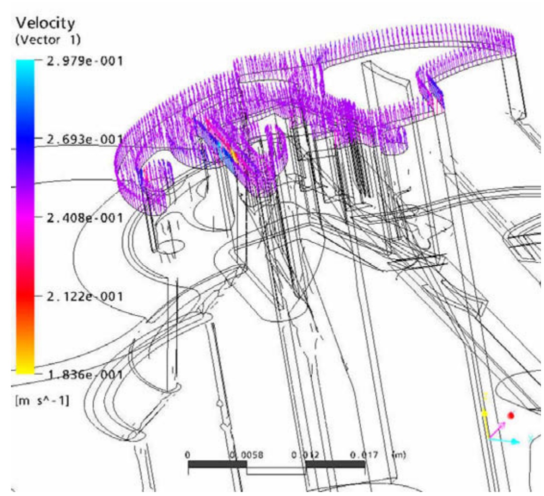

Fig. 3. Velocity vector diagram in the nozzle's outlet.

2.1. Experimental parameters

From different tests carried out by Metzeler Company in a rubber's extrusion die, we have obtained the power-law parameters for Eq. (2) shown in Table 1.

3. Mathematical model

To simulate this non-isothermal extrusion process, we have used the finite volume method (FVM). The most important characteristic of the FVM is that the resulting solution satisfies the conservation of quantities such as mass, momentum and energy for any control volume as well as for the whole computational domain and for any number of control volumes. Therefore, the integral conservation laws are written for a discrete volume [5,6]:

$$\frac{\partial}{\partial t}\int_{\Omega} U \, d\Omega + \oint_{s} \vec{F} \cdot d\vec{S} = \int_{\Omega} Q \, d\Omega, \tag{3}$$

where U is a scalar quantity, $\vec{F}$ is the flux vector, and Q are the volume sources. The first step in the FVM is to divide the domain into a

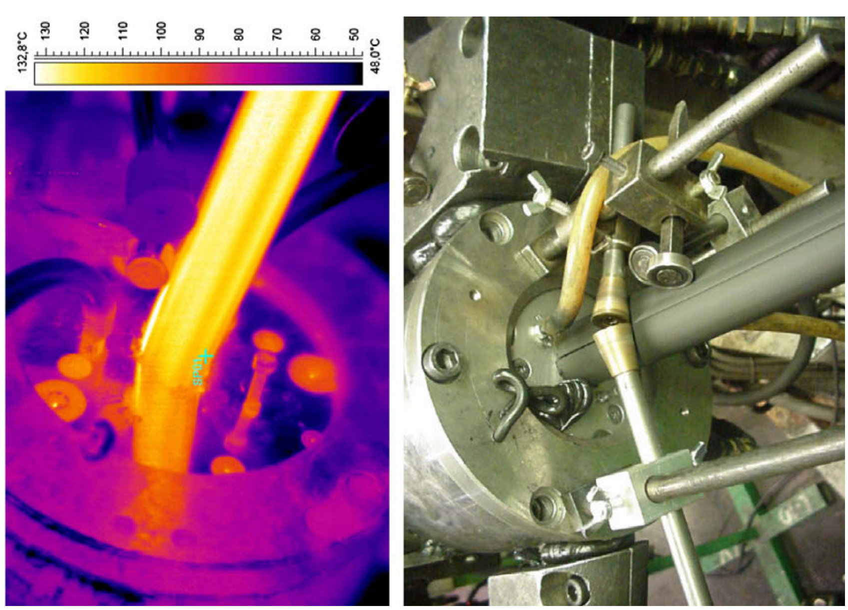

Fig. 2. Nozzle's outlet: temperature distribution (left) and real photograph of the rubber's extrusion die (right).

number of control volumes Ω_J (see Fig. 1) where the variable of interest is located at the centroid of the control volume. Eq. (3) is replaced by the discrete form:

$$\frac{\partial}{\partial t}(U_J\Omega_J) + \sum_{\text{sides}}(\vec{F}\cdot\vec{S}) = Q_J\Omega_J, \quad (4)$$

where the sum of the flux terms refers to all external sides of the control cell Ω_J. The values of the U_J variable are obtained by means of interpolation from the sides values.

4. Results and conclusions

In this work we have used a tetrahedral mesh with an element size of 0.0005 m. Our solver can be run so as to preserve time accuracy or as a pseudo-unsteady formulation to enhance convergence to steady state. From the experimental measurements it is observed that the velocity in the nozzle's outlet ranges from 0.15 to 0.35 m/s and from 0.18 to 0.30 m/s for the numerical results. It is also observed that the temperature distribution varies from 65 °C in the extrusion die to 120 °C in the rubber (Fig. 2). The variation of the velocity field (Fig. 3) is similar in both methods as well as the temperature field. Therefore, there is a good agreement between FVM numerical results and experimental test.

Acknowledgment

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